

## AP Environmental Science Math Prep

This year in APES you will hear the two words most dreaded by high school students...NO CALCULATORS! That's right, you cannot use a calculator on the AP Environmental Science exam. Since the regular tests you will take are meant to help prepare you for the APES exam, you will not be able to use calculators on regular tests all year either. The good news is that most calculations on the tests and exams are written to be fairly easy calculations and to come out in whole numbers or to only a few decimal places. The challenge is in setting up the problems correctly and knowing enough basic math to solve the problems. With practice, you will be a math expert by the time the exam rolls around. So bid your calculator a fond farewell, tuck it away so you won't be tempted, and start sharpening your math skills!

### Contents of this packet

Decimals

Averages

Percentages

Metric Units

Scientific Notation

Dimensional Analysis

### Reminders

1. **Write out all your work**, even if it's something really simple. This is required on the APES exam so it will be required on all your assignments, labs, quizzes, and tests as well.
2. **Include units** in each step. You have likely been lazy with this in past science classes. Not so now...Your answers always need units and it's easier to keep track of them if you write them in every step.
3. **Check your work**. Go back through each step to make sure you didn't make any mistakes in your calculations. Also check to see if your answer makes sense. For example, a person probably will not eat 13 million pounds of meat in a year. If you get an answer that seems unlikely, it probably is. Go back and check your work.

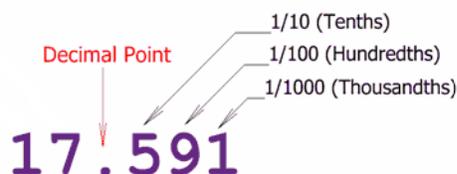
### Directions

Read each section below for review. Look over the examples and use them for help on the practice problems. When you get to the practice problems, write out all your work and be sure to include units on each step. Check your work.

## Decimals

### Part I: The basics

Decimals are used to show fractional numbers. The first number behind the decimal is the tenths place, the next is the hundredths place, the next is the thousandths place. Anything beyond that should be changed into scientific notation (which is addressed in another section.)



### Part II: Adding or Subtracting Decimals

To add or subtract decimals, make sure you line up the decimals and then fill in any extra spots with zeros. Add or subtract just like usual. Be sure to put a decimal in the answer that is lined up with the ones in the problem.

$$\begin{array}{r} 123.0000 \\ 0.0079 \\ +43.5000 \\ \hline 166.5079 \end{array} \qquad \begin{array}{r} 27.583 \\ - 0.200 \\ \hline 27.383 \end{array}$$

### Part III: Multiplying Decimals

Line up the numbers just as you would if there were no decimals. DO NOT line up the decimals. Write the decimals in the numbers but then ignore them while you are solving the multiplication problem just as you would if there were no decimals at all. After you have your answer, count up all the numbers behind the decimal point(s). Count the same number of places over in your answer and write in the decimal.

$$3.77 \times 2.8 = ?$$

$$\begin{array}{r} 3.77 \text{ (2 decimal places)} \\ \times 2.8 \text{ (1 decimal place)} \\ \hline 3016 \\ +754 \\ \hline 10.556 \text{ (3 decimal places)} \end{array}$$

### Part IV: Dividing Decimals

*Scenario One:* If the divisor (the number after the / or before the  $\overline{\hspace{1cm}}$ ) does not have a decimal, set up the problems just like a regular division problem. Solve the problem just like a regular division problem. When you have your answer, put a decimal in the same place as the decimal in the dividend (the number before the / or under the  $\overline{\hspace{1cm}}$ ).

$$\begin{array}{r} 424.9 \\ 38 \overline{) 16146.2} \\ \underline{152} \phantom{.2} \\ 94 \phantom{.2} \\ \underline{76} \phantom{.2} \\ 186 \phantom{.2} \\ \underline{152} \phantom{.2} \\ 342 \phantom{.2} \\ \underline{342} \phantom{.2} \\ 0 \end{array}$$

*Scenario Two:* If the divisor does have a decimal, make it a whole number before you start. Move the decimal to the end of the number, then move the decimal in the dividend the same number of places.

$$3.8 \overline{) 1614.62}$$

Then solve the problem just like a regular division problem. Put the decimal above the decimal in the dividend. (See Scenario One problem).

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

1.  $1.678 + 2.456 =$
2.  $344.598 + 276.9 =$
3.  $1229.078 + .0567 =$
4.  $45.937 - 13.43 =$
5.  $199.007 - 124.553 =$
6.  $90.3 - 32.679 =$
7.  $28.4 \times 9.78 =$
8.  $324.45 \times 98.4 =$
9.  $1256.93 \times 12.38 =$
10.  $64.5 / 5 =$
11.  $114.54 / 34.5 =$
12.  $3300.584 / 34.67 =$

## Averages

To find an average, add all the quantities given and divide the total by the number of quantities.

*Example:* Find the average of 10, 20, 35, 45, and 105.

*Step 1:* Add all the quantities.  $10 + 20 + 35 + 45 + 105 = 215$

*Step 2:* Divide the total by the number of given quantities.  $215 / 5 = 43$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

13. Find the average of the following numbers: 11, 12, 13, 14, 15, 23, and 29
14. Find the average of the following numbers: 124, 456, 788, and 343
15. Find the average of the following numbers: 4.56, .0078, 23.45, and .9872

## Percentages

### Introduction:

Percents show fractions or decimals with a denominator of 100. Always move the decimal TWO places to the right to go from a decimal to a percentage or TWO places to the left to go from a percent to a decimal.

*Examples:*  $.85 = 85\%$ .       $.008 = .8\%$

### Part I: Finding the Percent of a Given Number

To find the percent of a given number, change the percent to a decimal and MULTIPLY.

*Example:* 30% of 400

*Step 1:*  $30\% = .30$

*Step 2:* 400

x .30

12000

*Step 3:* Count the digits behind the decimal in the problem and add decimal to the answer.

$12000 \rightarrow 120.00 \rightarrow 120$

## **Part II: Finding the Percentage of a Number**

To find what percentage one number is of another, divide the first number by the second, then convert the decimal answer to a percentage.

*Example:* What percentage is 12 of 25?

*Step 1:*  $12/25 = .48$

*Step 2:*  $.48 = 48\%$  (12 is 48% of 25)

## **Part III: Finding Percentage Increase or Decrease**

To find a percentage increase or decrease, first find the percent change, then add or subtract the change to the original number.

*Example:* Kindles have dropped in price 18% from \$139. What is the new price of a Kindle?

*Step 1:*  $\$139 \times .18 = \$25$

*Step 2:*  $\$139 - \$25 = \$114$

## **Part IV: Finding a Total Value**

To find a total value, given a percentage of the value, DIVIDE the given number by the given percentage.

*Example:* If taxes on a new car are 8% and the taxes add up to \$1600, how much is the new car?

*Step 1:*  $8\% = .08$

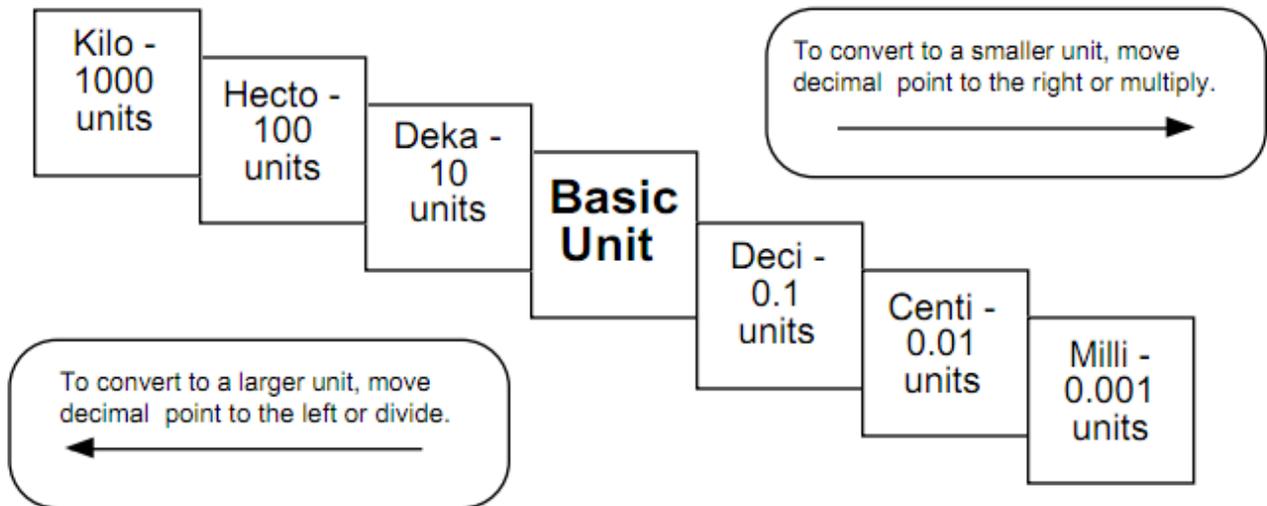
*Step 2:*  $\$1600 / .08 = \$160,000 / 8 = \$20,000$  (Remember when the divisor has a decimal, move it to the end to make it a whole number and move the decimal in the dividend the same number of places. .08 becomes 8, 1600 becomes 160000.)

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

16. What is 45% of 900?
17. Thirteen percent of a 12,000 acre forest is being logged. How many acres will be logged?
18. A water heater tank holds 280 gallons. Two percent of the water is lost as steam. How many gallons remain to be used?
19. What percentage is 25 of 162.5?
20. 35 is what percentage of 2800?
21. 14,000 acres of a 40,000 acre forest burned in a forest fire. What percentage of the forest was damaged?
22. You have driven the first 150 miles of a 2000 mile trip. What percentage of the trip have you traveled?
23. Home prices have dropped 5% in the past three years. An average home in Indianapolis three years ago was \$130,000. What's the average home price now?
24. The Greenland Ice Sheet contains 2,850,000 cubic kilometers of ice. It is melting at a rate of .006% per year. How many cubic kilometers are lost each year?
25. 235 acres, or 15%, of a forest is being logged. How large is the forest?
26. A teenager consumes 20% of her calories each day in the form of protein. If she is getting 700 calories a day from protein, how many calories is she consuming per day?
27. In a small oak tree, the biomass of insects makes up 3000 kilograms. This is 4% of the total biomass of the tree. What is the total biomass of the tree?

## Metric Units

Kilo-, centi-, and milli- are the most frequently used prefixes of the metric system. You need to be able to go from one to another without a calculator. You can remember the order of the prefixes by using the following sentence: *King Henry Died By Drinking Chocolate Milk*. Since the multiples and divisions of the base units are all factors of ten, you just need to move the decimal to convert from one to another.



*Example:* 55 centimeters = ? kilometers

*Step 1:* Figure out how many places to move the decimal. *King Henry Died By Drinking...* – that’s six places. (Count the one you are going to, but not the one you are on.)

*Step 2:* Move the decimal five places to the left since you are going from smaller to larger.

$$55 \text{ centimeters} = .00055 \text{ kilometers}$$

*Example:* 19.5 kilograms = ? milligrams

*Step 1:* Figure out how many places to move the decimal. ... *Henry Died By Drinking Chocolate Milk* – that’s six places. (Remember to count the one you are going to, but not the one you are on.)

*Step 2:* Move the decimal six places to the right since you are going from larger to smaller. In this case you need to add zeros.

$$19.5 \text{ kilograms} = 19,500,000 \text{ milligrams}$$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

28. 1200 kilograms = ? milligrams
29. 14000 millimeters = ? meters
30. 670 hectometers = ? centimeters
31. 6544 liters = ? milliliters
32. .078 kilometers = ? meters
33. 17 grams = ? kilograms

# Scientific Notation

## Introduction:

Scientific notation is a shorthand way to express large or tiny numbers. Since you will need to do calculations throughout the year WITHOUT A CALCULATOR, we will consider anything over 1000 to be a large number. Writing these numbers in scientific notation will help you do your calculations much quicker and easier and will help prevent mistakes in conversions from one unit to another. Like the metric system, scientific notation is based on factors of 10. A large number written in scientific notation looks like this:

$$1.23 \times 10^{11}$$

The number before the x (1.23) is called the coefficient. The coefficient must be greater than 1 and less than 10. The number after the x is the base number and is always 10. The number in superscript (11) is the exponent.

## Part I: Writing Numbers in Scientific Notation

To write a large number in scientific notation, put a decimal after the first digit. Count the number of digits after the decimal you just wrote in. This will be the exponent. Drop any zeros so that the coefficient contains as few digits as possible.

*Example: 123,000,000,000*

*Step 1: Place a decimal after the first digit. 1.23000000000*

*Step 2: Count the digits after the decimal...there are 11.*

*Step 3: Drop the zeros and write in the exponent.  $1.23 \times 10^{11}$*

Writing tiny numbers in scientific notation is similar. The only difference is the decimal is moved to the left and the exponent is a negative. A tiny number written in scientific notation looks like this:

$$4.26 \times 10^{-8}$$

To write a tiny number in scientific notation, move the decimal after the first digit that is not a zero. Count the number of digits before the decimal you just wrote in. This will be the exponent as a negative. Drop any zeros before or after the decimal.

*Example: .0000000426*

*Step 1: 00000004.26*

*Step 2: Count the digits before the decimal...there are 8.*

*Step 3: Drop the zeros and write in the exponent as a negative.  $4.26 \times 10^{-8}$*

## **Part II: Adding and Subtracting Numbers in Scientific Notation**

To add or subtract two numbers with exponents, the exponents must be the same. You can do this by moving the decimal one way or another to get the exponents the same. Once the exponents are the same, add (if it's an addition problem) or subtract (if it's a subtraction problem) the coefficients just as you would any regular addition problem (review the previous section about decimals if you need to). The exponent will stay the same. Make sure your answer has only one digit before the decimal – you may need to change the exponent of the answer.

*Example:*  $1.35 \times 10^6 + 3.72 \times 10^5 = ?$

*Step 1: Make sure both exponents are the same. It's usually easier to go with the larger exponent so you don't have to change the exponent in your answer, so let's make both exponents 6 for this problem.*

$$3.72 \times 10^5 \rightarrow .372 \times 10^6$$

*Step 2: Add the coefficients just as you would regular decimals. Remember to line up the decimals.*

$$\begin{array}{r} 1.35 \\ + .372 \\ \hline 1.722 \end{array}$$

*Step 3: Write your answer including the exponent, which is the same as what you started with.*

$$1.722 \times 10^6$$

## **Part III: Multiplying and Dividing Numbers in Scientific Notation**

To multiply exponents, multiply the coefficients just as you would regular decimals. Then add the exponents to each other. The exponents DO NOT have to be the same.

*Example:*  $1.35 \times 10^6 \times 3.72 \times 10^5 = ?$

*Step 1: Multiply the coefficients.*

$$\begin{array}{r} 1.35 \\ \times 3.72 \\ \hline 270 \\ 9450 \\ 40500 \\ \hline 50220 \end{array} \rightarrow 5.022$$

*Step 2: Add the exponents.*

$$5 + 6 = 11$$

*Step 3: Write your final answer.*

$$5.022 \times 10^{11}$$

To divide exponents, divide the coefficients just as you would regular decimals, then subtract the exponents. In some cases, you may end up with a negative exponent.

*Example:*  $5.635 \times 10^3 / 2.45 \times 10^6 = ?$

*Step 1: Divide the coefficients.*

$$5.635 / 3.45 = 2.3$$

*Step 2: Subtract the exponents.*

$$3 - 6 = -3$$

*Step 3: Write your final answer.*

$$2.3 \times 10^{-3}$$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet.

*Write the following numbers in scientific notation:*

34. 145,000,000,000
35. 13 million
36. 435 billion
37. .000348
38. 135 trillion
39. 24 thousand

*Complete the following calculations:*

40.  $3 \times 10^3 + 4 \times 10^3$
41.  $4.67 \times 10^4 + 323 \times 10^3$
42.  $7.89 \times 10^{-6} + 2.35 \times 10^{-8}$
43.  $9.85 \times 10^4 - 6.35 \times 10^4$
44.  $2.9 \times 10^{11} - 3.7 \times 10^{13}$
45.  $1.278 \times 10^{-13} - 1.021 \times 10^{-10}$
46. three hundred thousand plus forty-seven thousand
47. 13 million minus 11 thousand
48.  $1.32 \times 10^8 \times 2.34 \times 10^4$
49.  $3.78 \times 10^3 \times 2.9 \times 10^2$
50. three million times eighteen thousand
51. one thousandth of seven thousand
52. eight ten-thousandths of thirty-five million
53.  $3.45 \times 10^9 / 2.6 \times 10^3$
54.  $1.98 \times 10^{-4} / 1.72 \times 10^{-6}$
55. twelve thousand divided by four thousand

# Dimensional Analysis

## Introduction

Dimensional analysis is a way to convert a quantity given in one unit to an equal quantity of another unit by lining up all the known values and multiplying. It is sometimes called factor-labeling. The best way to start a factor-labeling problem is by using what you already know. In some cases you may use more steps than a classmate to find the same answer, but it doesn't matter. Use what you know, even if the problem goes all the way across the page!

In a dimensional analysis problem, start with your given value and unit and then work toward your desired unit by writing equal values side by side. Remember you want to cancel each of the intermediate units. To cancel a unit on the top part of the problem, you have to get the unit on the bottom. Likewise, to cancel a unit that appears on the bottom part of the problem, you have to write it in on the top.

Once you have the problem written out, multiply across the top and bottom and then divide the top by the bottom.

*Example: 3 years = ? seconds*

*Step 1: Start with the value and unit you are given. There may or may not be a number on the bottom.*

$$\left[ \frac{3 \text{ years}}{1} \right]$$

*Step 2: Start writing in all the values you know, making sure you can cancel top and bottom. Since you have years on top right now, you need to put years on the bottom in the next segment. Keep going, canceling units as you go, until you end up with the unit you want (in this case seconds) on the top.*

$$\left[ \frac{3 \cancel{\text{years}}}{1} \right] \left[ \frac{365 \cancel{\text{days}}}{1 \cancel{\text{year}}} \right] \left[ \frac{24 \cancel{\text{hours}}}{1 \cancel{\text{day}}} \right] \left[ \frac{60 \cancel{\text{minutes}}}{1 \cancel{\text{hour}}} \right] \left[ \frac{60 \text{ seconds}}{1 \cancel{\text{minute}}} \right]$$

*Step 3: Multiply all the values across the top. Write in scientific notation if it's a large number. Write units on your answer.*

$$3 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^7 \text{ seconds}$$

*Step 4: Multiply all the values across the bottom. Write in scientific notation if it's a large number. Write units on your answer if there are any. In this case everything was cancelled so there are no units.*

$$1 \times 1 \times 1 \times 1 = 1$$

*Step 5: Divide the top number by the bottom number. Remember to include units.*

$$9.46 \times 10^7 \text{ seconds} / 1 = 9.46 \times 10^7 \text{ seconds}$$

*Step 6: Review your answer to see if it makes sense.  $9.46 \times 10^7$  is a really big number. Does it make sense for there to be a lot of seconds in three years? YES! If you had gotten a tiny number, then you would need to go back and check for mistakes.*

In lots of APES problems, you will need to convert both the top and bottom unit. Don't panic! Just convert the top one first and then the bottom.

*Example: 50 miles per hour = ? feet per second*

*Step 1: Start with the value and units you are given. In this case there is a unit on top and on bottom.*

$$\left[ \frac{50 \text{ miles}}{1 \text{ hour}} \right]$$

*Step 2: Convert miles to feet first.*

$$\left[ \frac{50 \cancel{\text{ miles}}}{1 \text{ hour}} \right] \quad \left[ \frac{5280 \text{ feet}}{1 \cancel{\text{ mile}}} \right]$$

*Step 3: Continue the problem by converting hours to seconds.*

$$\left[ \frac{50 \cancel{\text{ miles}}}{1 \cancel{\text{ hour}}} \right] \quad \left[ \frac{5280 \text{ feet}}{1 \cancel{\text{ mile}}} \right] \quad \left[ \frac{1 \cancel{\text{ hour}}}{60 \cancel{\text{ minutes}}} \right] \quad \left[ \frac{1 \cancel{\text{ minute}}}{60 \text{ seconds}} \right]$$

*Step 4: Multiply across the top and bottom. Divide the top by the bottom. Be sure to include units on each step. Use scientific notation for large numbers.*

$$\begin{aligned} 50 \times 5280 \text{ feet} \times 1 \times 1 &= 264000 \text{ feet} \\ 1 \times 1 \times 60 \times 60 \text{ seconds} &= 3600 \text{ seconds} \\ 264000 \text{ feet} / 3600 \text{ seconds} &= 73.33 \text{ feet/second} \end{aligned}$$

Practice: Remember to show all your work, include units if given, and NO CALCULATORS! All work and answers go on your answer sheet. Use scientific notation when appropriate.

**Conversions that you may not be familiar with:**

1 square mile (mi<sup>2</sup>) = 640 acres

1 metric ton = 1000 kg

1 hectare (Ha) = 2.47 acres

1 ton = 2000 lb.

1 kilowatt-hour (kWh) = 3,413 BTUs

1 lb. = 16 oz.

1 barrel of oil = 160 liters

1 Km = .62 mi.

56. 134 miles = ? inches

57.  $8.9 \times 10^5$  tons = ? ounces

58. 1.35 kilometers per second = ? miles per hour

59. A city that uses ten billion BTUs of energy each month is using how many kilowatt-hours of energy?

60. A 340 million square mile forest is how many hectares?

61. If one barrel of crude oil provides six million BTUs of energy, how many BTUs of energy will one liter of crude oil provide?

62. Fifty eight thousand kilograms of solid waste is equivalent to how many metric tons?